COMPETITION COMMISSION

COORDINATED EFFECTS MERGER SIMULATION WITH LINEAR DEMANDS

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Abstract

This paper provides an example of a methodology for evaluating the potential 'coordinated effects' of mergers in differentiated product markets. Specifically, I examine coordinated effects merger simulation when demand systems are linear and marginal costs are constant. I show that these assumptions are sufficient to ensure that analytic solutions for the required 'collusive,' 'Nash' and 'defection' pricing strategies are available. Consequently the methodology outlined here is easy for antitrust authorities (and others) to implement. A full implementation of the methodology using data from the Network server market and 'best practice' demand structures is provided in Davis, Huse and Van Reenen (2006). Here I provide numerical examples, illustrating the techniques and demonstrating the 'coordinated effects' of simulated mergers. While the numerical results are special to the particular parameters of the demand system considered, the analytic results ensure that the calculations can easily be performed for essentially any linear demand structure and any ownership structure in the market. The numerical examples demonstrate clearly that mergers may enhance the likelihood of collusion but they also show that mergers will sometimes make collusion more difficult to sustain, in particular when mergers create highly asymmetric market structures. Along the way I show (i) that a folk theorem does not hold in general differentiated product games, and (ii) that it can be the small firm who is hardest to induce to collude.

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1. Introduction

This paper reports on a methodology for empirically determining whether firms have incentives to tacitly collude in differentiated product markets. The techniques I illustrate here will be particularly useful in merger control where policymakers must decide whether or not to allow proposed mergers. Currently, merger control authorities can and do block mergers on the basis that the smaller number of firms post-merger may make tacit collusion more likely. In merger evaluation, this is known as the theory of 'coordinated effects.' Indeed coordinated effects were the primary reason given for blocking mergers until at least the early 1990's when 'unilateral effects,' the idea that when firms producing close substitutes merge static equilibrium prices may go up, also emerged as a major source of concern. Recent examples in which antitrust authorities have invoked the theory of coordinated effects include the *Nestle-Perrier*, *Kali and Salz*, *Gencor-Lenrho* and *Airtours* cases in the EU jurisdiction and *Safeway* in the UK. (See Dick (2003) for a discussion of the large number of recent coordinated effects cases in the US.)

When blocking mergers on the grounds of 'coordinated effects', the authorities follow a rich theoretical economics literature emphasizing that oligopolists who meet regularly in the marketplace may tacitly collude on higher prices. Chamberlin (1929) argued this point informally while Stigler (1964), Friedman (1971) and a large number of subsequent authors formalized this intuition in the theory of repeated games (see Aumann (1986,1989) and Mertens (1987) for surveys.) I merge this theoretical literature on repeated games with the empirical literature on the analysis of pricing games in differentiated product markets. In particular, I use the recent literature on the evaluation of the 'unilateral' effects of mergers (Werden and Froeb (1994), Hausman et al (1994) and Nevo (2001)). In doing so I hope to provide techniques which will be useful supplements to the quantitative techniques currently used in practice to evaluate coordinated effects; see Scheffman and Coleman (2003).

In order to decide whether a merger would result in an increased likelihood of tacit collusion, antitrust authorities proceed by considering the presence or absence of conditions that facilitate collusion. Specifically, in order to sustain collusion firms must be able to (i) come to an agreement (which can be difficult when products are complex and differentiated), (ii) monitor each others' behavior (in order to detect cheaters) and (iii) enforce collusive behavior collectively by punishing those incumbent firms who cheat (internal stability) and deterring entry by new potential rivals or expansion by a competitive fringe (external stability).

In this paper, we examine when firms would be able to sustain tacitly collusive outcomes using the most basic enforcement mechanism suggested by the theory of repeated games, 'grim' strategies. When using a 'grim' strategy, a firm plays a collusive action to begin with and continues to do so as long as (s)he never detects that a rival has cheated. If cheating is detected, she plays the static Nash equilibrium strategy in all subsequent periods. Grim strategies provide an enforcement mechanism for tacitly colluding firms because they punish cheating against a collusive arrangement by ensuring that a cheater will sacrifice their share of future collusive profits in return for short run gains today. Provided firms are sufficiently patient, Friedman (1971) showed that, in homogeneous product settings, grim strategies will suffice to sustain tacitly collusive equilibria. This result is known as Friedman's 'folk theorem.' One contribution of this paper is to provide an example demonstrating that, for grim strategies, such a folk theorem does not apply in differentiated product markets.

In sum, our aim in this paper is to take seriously the idea that dynamic oligopoly models can be taken to data in differentiated product markets. As Kuhn and Motta (2004) note, the judgment by the Court of First Instance in annulling the decision of the European Commission to block the *Airtours* merger has clarified that 'joint dominance' analysis in European mergers has to be treated as coordinated effects analysis and thus has to be consistent with collusion theory. Thus, our aim in this paper is to provide a natural 'next step' in developing merger simulation into a practical toolbox, one that can effectively allow competition authorities to consider whether competition concerns arise from either potential unilateral or potential coordinated effects of mergers.

The methods presented here are most directly useful to evaluate (i) internal stability and (ii) one element of external stability; the incentive for the competitive fringe to expand in the face of a cartel. Simulation of the other key element of external stability, whether cartel profits would attract new entrants to the industry, requires additional information, for example an evaluation of the costs of entry into the industry, and would involve a non-trivial extension of the model.

Our work is related to a number of prior literatures. The most directly related empirical literature has attempted to evaluate the existing conduct of firms using game-theoretic pricing models. Specifically, authors following Gollop and Roberts (1979), Bresnahan (1982), Lau (1982), Roberts (1983), Porter (1983), Suslow (1986), Bresnahan (1982,1987), Gasmi, Laffont and Vuong (1990, 1992), Nevo (1998, 2001), Slade (2004) and Salvo (2004) have attempted to evaluate whether observed equilibrium prices are more consistent with collusive or Nash equilibrium pricing.[2] In contrast to this branch of the literature, I focus primarily on evaluating whether collusion is sustainable as an equilibrium.

In the theoretical literature, some recent related theoretical contributions have begun to study collusion in settings with asymmetric market structures. The fact that the analysis must consider cases where firms are asymmetric post-merger makes theoretical analysis in this area challenging. Nonetheless, there is some recent progress. In particular, Compte, Jenny and Rey (2002) examine coordinated effects in the context of a Bertrand-Edgeworth homogeneous goods model with capacity constraints and calibrate their model with the data from the *Nestle-Perrier* case. In their model, capacity constraints mean it is the large firms who can both be tempted to cheat and also have the ability to punish their rivals. Thus, it is the firm with large capacities who must be induced to collude, perhaps by giving smaller firms spare capacity and thus an ability to punish their larger rivals. We will find that the opposite can also be true—that the small firms can be the ones who will be difficult to induce to collude. The reason is fundamentally the standard observation from merger models in differentiated product markets—that small firms like concentrated rivals because the larger or more collusively their rivals act, the higher the prices charged in Nash equilibrium. That fact however makes it difficult to induce small firms to collude, because their share of the collusive pie (at least in a world without side payments) will be small while their freedom to undercut their rivals' prices is more valuable the more concentrated their rivals are.

Kuhn and Motta (2004) provide the first paper to study asset transfers in differentiated product markets. The intuition for their conclusions regarding asymmetry is identical to

[2]Salvo (2004) provides the most recent contribution to the literature, examining the determination of conduct when the potential for import substitution constrains the behaviour of incumbent firms to charge high prices.

ours while their setting is far more complex—it is one of imperfect information. Unfortunately, to study asset transfers theoretically they must make rather strong assumptions, ones that make their model difficult to consider directly as a basis for empirical work. For example they consider only the case when the price for every good sold by each firm is the same.[3] For that reason, the numerical and simulation approach followed in this paper seems likely to be useful.

The closest paper to our own is provided by Sabbatini (2004) of the Italian Antitrust Authority who has recently, and independently, suggested a similar approach to coordinated effects merger simulation.[4]

Throughout the paper I take the stance that merger simulation is a useful and interesting exercise. Some authors warn that simulation results are often sensitive to the details of the model (see for example the discussion in Walker (2005).) Such observations however are not necessarily critiques—we actively want merger investigations to have different outcomes depending on the nature of, say, substitution patterns between goods. In any modeling exercise, one must always be careful to specify the empirical and theoretical components of the model appropriately and merger simulation exercises are no different in this regard. The good news is that given enough data, we will be able to tell models apart from one another given the right data variation (see in particular Bresnahan (1982) and Nevo (1998).) Even without rich datasets, simulation allows investigators to describe the way in which their conclusions are, or are not, sensitive to baseline modeling assumptions. A range of plausible scenarios can therefore be evaluated.

In the case of coordinated effects merger simulation we have expanded the set of models of firm behaviour being considered, from static to dynamic price setting models. While extending the set of models being considered is advantageous, it does not come without a price. For example, there are numerous alternatives to using grim strategies to sustain collusive arrangements. The good news is that the techniques outlined here can, at least in principle, be amended to also allow their evaluation. In particular, the results provided in Abreu (1988) provide an algorithm for checking whether more sophisticated 'simple penal codes' are sub-game perfect Nash equilibrium strategies. On the other hand, we choose to focus on grim strategies because they are simplest, are well understood, do not assume that companies can punish optimally and will generally provide a coherent benchmark against which to judge whether there are likely to be increased incentives for tacit collusion. However, we do not pretend that this is the last word on the topic and we intend to return to more sophisticated treatments in subsequent papers.

An area of particular concern in model specification is the underlying demand system (see Walker (2005)). Again, given enough data, such modeling choices can usually be tested against one another. Linear and log-linear demand specifications for example can be distinguished using a 'Box-Cox' test (see any good econometrics textbook.) This paper studies only the case in which demand curves are linear and marginal costs are

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[3]Their paper is, however, richer than this one in the sense that they study collusion under asymmetric information following the papers by Abreu, Pearce and Stachetti (1986, 1990).
[4]I thank Mark Ivaldi for bringing Sabbatini's interesting working paper to my attention. Pierluigi Sabbatini and I anticipate combining our respective research efforts into a single next version of our paper(s) if we can agree a common 'best practice' approach. At present, Sabbatini favours a notion due to Friedman (1971) called a 'balanced temptation equilibrium,' a related but somewhat different approach to that outlined here.

constant in output. For this special class of models, analytic results are derived for each component of the model, facilitating computation enormously. The techniques are illustrated here using a particular and simple numerical example. A full implementation of our 'coordinated effects' methodology using data from a real market, the Network Server (computer) market, as well as 'best practice' demand structures is provided in Davis, Huse and Van Reenen (2006). In that paper, we use random coefficient discrete choice demand structures following Berry (1994), Berry, Levinsohn and Pakes (1995) and Nevo (2001).

The paper proceeds as follows. Section 2 develops the single period game that we assume is played by our firms and introduces a numerical example. The material in this section will be familiar from the unilateral effects merger simulation literature. Section 3 introduces the dynamic game and shows that, to analyze it, we need only calculate one additional item beyond those required for the 'unilateral effects' merger simulation literature, namely the payoff to 'defection'. Section 4 continues the numerical example, presenting some 'coordinated effects' merger simulations. In particular, an example is provided that demonstrates that when market structures are made more asymmetric by a merger, collusion can become more difficult to sustain after a merger, rather than less. These numerical results are therefore consistent with the recent theoretical results provided by Compte, Jenny and Rey (2002) and Kuhn and Motta (2004) and add weight to those arguing against a strict structural (market share or Herfindahl) based test for an evaluation of whether a merger increases the likelihood of collusion. In section 5 I conclude and suggest some directions for future research.

2. Unilateral effects and the single period game

This section presents the stage-game of the dynamic model. The stage game is simply a standard differentiated product Bertrand pricing game, identical to that used in the unilateral effects merger simulation literature by Werden and Froeb (1991), Berry (1994), Hausman et al (1994), Berry, Levinsohn and Pakes (1995) and Nevo (2001). In particular, I consider the case of linear demand systems; this facilitates the provision of analytic solutions for use in undertaking both unilateral and coordinated effects merger simulations. Doing so provides a set of results for antitrust authorities that are particularly simple to implement.

Specifically, suppose that demand for product $k \in \{1,..,J\} \equiv \Im$ may be written as a linear function of the prices of all the goods in the market:

$$D_k(p_1, p_2, ..., p_J) = \begin{cases} a_k + \sum_{j=1}^{J} b_{kj} p_j & \text{if} \quad a_k + \sum_{j=1}^{J} b_{kj} p_j \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

with demand intercept parameter a_k and slope parameter b_{kj} describing the change in demand for product k when good j's price increases by £1. Consider the pricing game wherein each firm *f* produces a subset of the available products, $\Im_f \subseteq \Im$, and chooses the prices of those products to maximize its profits:

$$\max_{\{p_j | j\in\Im_f\}} \sum_{j\in\Im_f} (p_j - c_j) D_j(p)$$
$$s.t. \quad p_j \geq 0 \qquad for \quad j \in \Im_f$$

where $p = (p_1, p_2, ..., p_J)$ and c_j is the marginal cost of product j, assumed constant. Notice that for the linear demand and constant marginal cost case, this objective function is a quadratic function of prices while the constraints are linear functions of prices so that the problem may easily be solved numerically using standard optimization tools for quadratic programs.[5] Alternatively, provided equilibrium prices of all goods in the market are positive and all goods are sold in positive quantities as is universally assumed in the existing empirical literature (and so the constraints for this program do not bind in equilibrium), we may solve this problem analytically by examining the first order conditions to the unconstrained problem:

$$D_k(p) + \sum_{j\in\Im_f} \frac{\partial D_j(p)}{\partial p_k}(p_j - c_j) = 0 \text{, for all } k \in \Im_f$$

where, with linear demands, $\frac{\partial D_j(p)}{\partial p_k} = b_{jk}$.

At this point, the literature on unilateral effects games has found it useful to introduce an 'ownership matrix' to standardize these first order conditions. Specifically, define the (JxJ) matrix Δ with j,k[th] element:

$$\Delta_{kj} = \begin{cases} 1 & j,k \text{ produced by same firm} \\ 0 & \text{otherwise} \end{cases}$$

where, by construction $\Delta_{kj} = \Delta_{jk}$ for all $j,k \in \Im$. Notice that changing the ownership structure in unilateral effects merger simulations reduces solely to changing this ownership indicator matrix. Using the ownership indicators, firm *f*'s first order condition may be simply rewritten as:

$$D_k(p) + \sum_{j=1}^{J} \Delta_{jk} \frac{\partial D_j(p)}{\partial p_k}(p_j - c_j) = 0 \quad \text{ie,} \quad a_k + \sum_{j=1}^{J} b_{kj} p_j + \sum_{j=1}^{J} \Delta_{jk} b_{jk} (p_j - c_j) = 0 \quad (1)$$

Notice that there is one of these first order conditions from firm *f*'s objective function for every $k \in \Im_f$. Since every product is owned by some firm, under the behavioural assumptions that each firm prices its products to maximize its profits from the stage game, we obtain a total of J first order conditions—one for every product. We may then 'stack up' the first order conditions. To do so it is useful to introduce some matrix notation. Define the (Jx1) vector $a = (a_1, ..., a_J)'$ and also the matrices:

[5]Matlab, Gauss, Mathematica or Maple can all solve this kind of problem easily and quickly. Constraints requiring that equilibrium prices and quantities be non-negative may also be added easily using those quadratic programming tools.

$$B' = \begin{bmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1J} \\ \vdots & & \vdots & & \vdots \\ b_{k1} & \cdots & b_{kj} & \cdots & b_{kJ} \\ \vdots & & \vdots & & \vdots \\ b_{J1} & \cdots & b_{Jj} & \cdots & b_{JJ} \end{bmatrix} \text{ and } \Delta \bullet B = \begin{bmatrix} \Delta_{11}b_{11} & \cdots & \Delta_{j1}b_{j1} & \cdots & \Delta_{J1}b_{J1} \\ \vdots & & \vdots & & \vdots \\ \Delta_{1k}b_{1k} & & \Delta_{jk}b_{jk} & & \Delta_{Jk}b_{Jk} \\ \vdots & & \vdots & & \vdots \\ \Delta_{1J}b_{1J} & \cdots & \Delta_{jJ}b_{jJ} & \cdots & \Delta_{JJ}b_{JJ} \end{bmatrix}$$

(where $\Delta \bullet B$ is known as the 'Hadamard' or 'element by element' or 'dot' product). In the Appendix I show that given these definitions, we can write the demand system for all goods as $D(p) = a + B'p$, while the vector of first order conditions can also be written very compactly as $a + B'p + (\Delta \bullet B)(p - c) = 0$.

The solution to this set of equations is the Nash equilibrium vector of prices, $p^{NE} = (p_1^{NE},..., p_J^{NE})$ since, by construction, each firm is choosing the prices of its products to maximize its profits given the prices charged by other firms. Rearranging the first order conditions gives an analytic matrix expression for the Nash equilibrium prices of all products:

$$p^{NE} = (B' + (\Delta \bullet B))^{-1}\left(-a + (\Delta \bullet B)c\right).$$

The beauty of the linear demand curve specification is that every object of interest can be computed easily for any ownership structure, Δ. For instance, given the expression for the Nash equilibrium prices above, equilibrium demands for each product are given by the vector $D(p^{NE}) = a + B'p^{NE}$ and profits derived from each *product* are $(p^{NE} - c) \bullet D(p^{NE})$ where $\bullet$ represents the element by element multiplication of the two Jx1 vectors $(p^{NE} - c)$ and demands $D(p^{NE}) = a + B'p^{NE}$ respectively. The profits of each *firm* can therefore be calculated by adding across the owned products. If firm f owns product j and $\Delta'_{j.}$ is the jth row of the ownership matrix Δ, then firm f's Nash equilbrium profits $\Pi_f(p^{NE})$ may be calculated using $\Pi_f(p^{NE}) = \Delta'_{j.}\left((p^{NE} - c) \bullet (a + B'p^{NE})\right)$.

2.1 Unilateral effects merger simulations

In a unilateral effects merger simulation, where firm *f* 's set of products and costs before and after the merger are respectively $\Im_f^{PRE}$ and $\Im_f^{POST}$, and c_j^{PRE} and c_j^{POST} for all $j \in \Im_f^{PRE}$ and $j \in \Im_f^{POST}$ respectively. These ownership structure changes are captured in the matrix Δ, so a 'unilateral effects' merger simulation with linear demands amounts to calculating the Nash equilibrium 'NE' prices in each case:

$$p^{NE,PRE} = (B' + (\Delta^{PRE} \bullet B))^{-1}\left(-a + (\Delta^{PRE} \bullet B)c^{PRE}\right)$$

and

$$p^{NE,POST} = (B' + (\Delta^{POST} \bullet B))^{-1}\left(-a + (\Delta^{POST} \bullet B)c^{POST}\right).$$

Naturally, often data will not be available on marginal costs. If not, then the method suggested by the unilateral effects literature is to estimate the marginal costs using pre-merger prices and an assumption, which may be tested, about the nature of price competition in the pre-merger period. Specifically, given the observed pre-merger prices p^{PRE} and the ownership matrix Δ^{PRE}, we may solve the first order conditions for pre-merger marginal costs:

$$c^{PRE} = (\Delta^{PRE} \bullet B)^{-1}\left(a + (B' + (\Delta^{PRE} \bullet B))p^{PRE}\right).$$

2.2 A numerical example:

In this subsection we will consider a unilateral effects merger simulation, but we will use the same example demand structure and cost assumptions throughout the paper. Specifically, consider the case where there are a total of six products j=1,2,...,6 in the market and where there are no efficiencies that result from the merger, so that $c_j^{PRE} = c_j^{POST} = 1$. Suppose further that the linear demand structure in a market and where the 1st product's demand equation is given by:

$$q_1 = 10 - 2p_1 + 0.3p_2 + 0.3p_3 + 0.3p_4 + 0.3p_5 + 0.3p_6$$

and the others, for j=2,..,6 are symmetrically defined with the coefficient –2 on own price and the coefficients 0.3 on rival products' prices. In our formulae derived above therefore, we set each $a_j = 10$ and construct the matrix B to have –2's along the diagonal and all off-diagonal elements take the value 0.3.

Table 1 reports the predicted static equilibrium prices under a variety of ownership structures. For example, the first column reports the ownership structure where every product is owned by a different firm while the sixth column reports the case where there is a single firm owning all six products. The intermediate columns report intermediate market structures so for example, (4,2) indicates that the first firm owns four products and the second firm owns two. Table 2 reports the resulting equilibrium profits.

TABLE 1 **Predicted static equilibrium prices for each product under a variety of market structures. The shaded cells represent the products produced by the largest firm under each ownership structure.**

	Market structure					
Product	*(1,1,1,1,1,1)*	*(2,2,2)*	*(3,3)*	*(4,2)*	*(5,1)*	*6(Cartel)*
1	4.8	5.3	5.9	6.6	7.9	10.5
2	4.8	5.3	5.9	6.6	7.9	10.5
3	4.8	5.3	5.9	6.6	7.9	10.5
4	4.8	5.3	5.9	6.6	7.9	10.5
5	4.8	5.3	5.9	5.8	7.9	10.5
6	4.8	5.3	5.9	5.8	6.0	10.5

TABLE 2 **Predicted static equilibrium profits for each firm under a variety of market structures**

	Market structure					
Firms	*(1,1,1,1,1,1)*	*(2,2,2)*	*(3,3)*	*(4,2)*	*(5,1)*	*6(Cartel)*
1	28.9	63.4	105.0	139.0	188.5	270.8
2	28.9	63.4	105.0	77.6	49.0	
3	28.9	63.4				
4	28.9					
5	28.9					
6	28.9					
Industry profits	173.0	190.0	210.0	217.0	238.0	270.8

3. The repeated game

The next step is to consider the above analysis as a stage game within the broader context of an infinitely repeated game. Following the repeated game literature, each firm is assumed to maximize the net present value (NPV) of its profits, and we require that at each point in the game tree the firm makes choices which are optimal given that it reached that node of the game tree so that we study sub-game perfect equilibria of the repeated game (Selten 1965).

Following Friedman (1971), we will consider the feasibility of sustaining a candidate collusive equilibrium using 'grim' strategies. Friedman (1971) demonstrated that if each player adopted 'grim' strategies, and was sufficiently patient, then there can be a large number of sub-game perfect equilibria of the dynamic game, sometimes including the outcome that firms choose to price in a way that maximizes industry profits in each period of the game, ie, each stage-game. In contrast to Friedman, who considered the homogeneous products case, we will consider the differentiated product game.

To do so, we must first introduce some notation. Denote the one period Nash equilibrium and collusive payoffs to firm *f* as π_f^{NE} and $\pi_f^{collusion}$ respectively. These are exactly the payoffs reported in Table 2 above. Similarly denote the one period gain to firm *f* from defection when all other firms are playing collusively as $\pi_f^{defection}$. We will discus how to compute $\pi_f^{defection}$ extensively in section 3.1 below.

When rivals are playing grim strategies a defector earns his one period defection payoff and then subsequently receives only his Nash equilibrium profits. Thus, the net anticipated return to defection today is $V_f^{defection}(\delta_f) = \pi_f^{defection} + \frac{\delta_f \pi_f^{NE}}{1-\delta_f}$ for firm *f*, while her payoff to collusion today and in all subsequent periods given that rivals continue to collude is given by $V_f^{Collusion}(\delta_f) = \frac{\pi_f^{collusion}}{1-\delta_f}$. Hence, player *f* has no incentive to deviate from collusive pricing provided that:

$$V_f^{collusion}(\delta_f) > V_f^{defection}(\delta_f) \quad \Leftrightarrow \quad \frac{\pi_f^{collusion}}{1-\delta_f} > \pi_f^{defection} + \frac{\delta_f \pi_f^{NE}}{1-\delta_f}.$$

In order to examine the incentives to collude using grim strategies, we must therefore consider the returns achieved by each firm in the three pricing scenarios—'collusion', 'Nash equilibrium pricing' and 'defection.' We have already demonstrated—using unilateral effects analysis—how to calculate Nash equilibrium profits and also the returns to (perfect) collusion. It remains therefore only to calculate the payoff to defection.

Before providing expressions for each component in this incentive compatibility constraint, we note that the *only* components in this equation that are not already evaluated in a unilateral effects merger simulation are (i) the payoff to defection $\pi_f^{defection}$ and (ii) the discount factor δ_f. The former, like the Nash and Collusive equilibrium payoffs, depends directly on the nature of the static profit function for each firm and therefore may be easily calculated using the methodologies developed for the analysis of data generated by static pricing games (see below).

In an antitrust case, the discount factor could usually be taken from internal documents specifying the company's required rate of return. Alternatively, if companies are listed, CAPM or another rate of return model could be used to infer an appropriate discount rate for payoffs. Thirdly, more closely paralleling the theoretical literature, we can report the range of discount factors for which collusion could be sustained under any given industry structure. Since I examine a numerical example, here I take the latter approach.

3.1 The payoff to defection

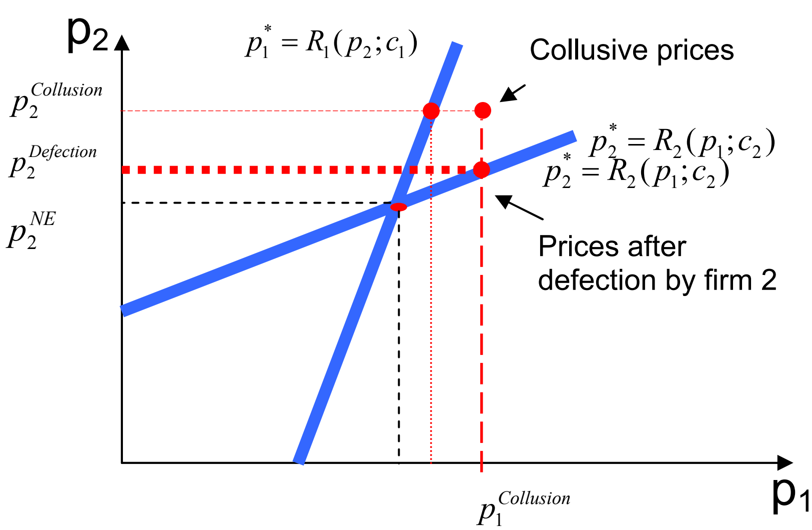
Following the theoretical literature on repeated games, define the payoff to firm *f* from defection as the maximum amount of profit that could be achieved given its rivals' prices (i.e. treating them as fixed). In the case most directly of interest, where firm f is deciding whether or not to defect from the tacitly collusive agreement, other firms will be choosing their prices to be the collusive prices, and so the static payoff to firm *f* when defecting is:

$$
\begin{aligned}
&\pi_f^{defection} \equiv \max_{\{p_j | j \in \Im_f\}} \sum_{j \in \Im_f} (p_j - c_j) D_j(\underline{p}_f, \underline{p}_{-f}^{collusion}) \\
&s.t. \\
&\qquad p_j \geq 0 \qquad and \\
&\qquad D_j(\underline{p}_f, \underline{p}_{-f}^{collusion}) \leq \overline{k}_j \quad for \quad j \in \Im_f
\end{aligned}
$$

Where $\underline{p}_f$ denotes the vector of prices of goods produced by firm f, $\underline{p}_{-f}^{collusion}$ denotes the collusive prices of other firms and where the last set of constraints enforce capacity constraints on the defecting firm. For the case of linear demand equations, this non-linear maximization problem is again a quadratic objective function subject to linear constraints and so is easy to solve numerically, even for large problems, using standard methods—such as the quadratic programming toolbox provided as a standard element within Matlab or Gauss. In other cases, it must be solved using more general non-linear optimization techniques but even in those cases since it involves only an optimization, it is a simpler mathematical object to evaluate than the Nash Equilibrium that must be computed in unilateral effects merger simulations. Figure 1 describes the calculation of a 'defection' price in the context of an example with two single product firms. Solving for the defection prices simply involves finding the location on the deviating firm's best response function at the point where its rival is charging collusive profits.

In the case where capacity constraints do not bind and prices and quantities are positive in equilibrium, we can provide an analytical solution to this problem when demand curves are linear for arbitrary ownership structures. To do so we derive the first order conditions for this problem, which are just those computed for a single firm in the 'unilateral effects' simulations evaluated when rival firms charge collusive prices.

FIGURE 1



Considers the case with two single product firms. Each line shows a firm's 'reaction function', $p_1^* = R_1(p_2;c_1)$ and $p_2^* = R_2(p_1;c_2)$ respectively, describing the price that maximizes the firm's profits, given the price charged by the rival firm. Where these reaction functions intersect describes the Nash equilibrium prices. Collusive prices, those which maximize industry profits are also described. Each of these prices is calculated in a unilateral effects merger simulation. The 'defection' price is that which maximizes firms profits given that the rival is charging the collusive price. Here that can be found for each player by evaluating their reaction function price when their rival is charging the collusive prices.

In fact, the first order condition for this problem with a linear demand system may be written in terms of matrices and this facilitates the provision of an analytic solution to this problem. To do so, it helps to introduce some notation. Define for any (JxJ) matrix A, the sub-matrix $A_{[f,f]}$ which corresponds to just the rows and columns of A from products owned by firm f. Similarly, define $A_{[f,-f]}$ to be the rows of A corresponding to products owned by firm f and the columns of A corresponding to products owned by other firms and the sub-matrix $A_{[f,.]}$ which means the rows of A for *f*'s products and all columns of A. Similarly, for any (Jx1) vector a, define a_f simply to be the rows of the vector a corresponding to products that firm f produces.

The first order conditions from the profit maximization problem that is defined above for the capacity unconstrained version of the 'defection problem' can be written as:

$$D_k(p)+\sum_{j=1}^{J}\Delta_{jk}\frac{\partial D_j(p)}{\partial p_k}(p_j-c_j)=0 \text{ for each } k\in\Im_f$$

evaluated at $p=(\underline{p}_f^{Defection},\underline{p}_{-f}^{collusive})$ where $\underline{p}_{-f}^{collusive}$ is parametric and $\underline{p}_f^{Defection}$ is the price which solves this system of first order equations. We can stack up these moment conditions and obtain the vector equation: $a_f+B^{'}_{[.,f]}p+(\Delta\bullet B)_{[f,.]}(p-c)=0$.[6]

Breaking up this expression so that we can write it in terms of prices of goods owned by firm f and prices of goods owned by rivals gives:

$$a_f-[\Delta\bullet B]_{[f,.]}c+\left(B^{'}_{[f,f]}+[\Delta\bullet B]_{[f,f]}\right)\underline{p}_f^{Defection}+\left(B^{'}_{[-f,f]}+[\Delta\bullet B]_{[f,-f]}\right)\underline{p}_{-f}^{Collusive}=0$$

which can in turn be rearranged to give:

$$\left(B^{'}_{[f,f]}+[\Delta\bullet B]_{[f,f]}\right)\underline{p}_f^{Defection}=-\left(a_f-[\Delta\bullet B]_{[f,.]}c\right)-\left(B^{'}_{[f,-f]}+[\Delta\bullet B]_{[f,-f]}\right)\underline{p}_{-f}^{Collusive}$$

and hence we can solve for firm f's defection prices analytically as:

$$\underline{p}_f^{Defection}=-\left(B^{'}_{[f,f]}+[\Delta\bullet B]_{[f,f]}\right)^{-1}\left(\left(a_f-[\Delta\bullet B]_{[f,.]}c\right)-\left(B^{'}_{[-f,f]}+[\Delta\bullet B]_{[f,-f]}\right)\right)\underline{p}_{-f}^{Collusive}.$$

Having provided the definitions of the three core elements of the incentive compatibility constraint, we are now in a position to summarize the methodology and progress to our numerical examples of coordinated effects merger simulations.

4. Coordinated effects merger simulations

In this section we will present a numerical example of our methodology for evaluating the 'coordinated effects' of mergers. Before doing so I summarize the methodology which involves taking the six steps reported in Table 3.

[6] Note that the notation $B^{'}_{[.,f]}$, denotes the matrix $B_{[.,f]}$ transposed. If the products are suitably ordered, we can write $B=\begin{bmatrix}B_{[.,f]} & B_{[.,-f]}\end{bmatrix}$ and $B_{[.,f]}=\begin{bmatrix}B_{[f,f]}\\ B_{[-f,f]}\end{bmatrix}$ so $B^{'}_{[.,f]}=\begin{bmatrix}B^{'}_{[f,f]} & B^{'}_{[-f,f]}\end{bmatrix}$.

TABLE 3 **The procedure to undertake a coordinated effects merger simulation**

Step no.	*Description*
1	Estimate the differentiated product demand system
2	Use the pre-merger period data to infer marginal costs for each product by using an appropriate assumption about the nature of pre-merger prices (usually that it is a static Bertrand—Nash equilibrium.)
3	Calculate the static Nash and collusive equilibrium prices and payoffs, $(\underline{p}_f^{NE}, \pi_f^{NE})$ and $(\underline{p}_f^{collusion}, \pi_f^{collusion})$ respectively
4	Calculate the 'defection' prices and profits, $(\underline{p}_f^{defection}, \pi_f^{defection})$
5	Evaluate each component of the incentive compatibility constraint:[7] $V_f^{collusion}(\delta_f) > V_f^{defection}(\delta_f) \Leftrightarrow \frac{\pi_f^{collusion}}{1-\delta_f} > \pi_f^{defection} + \frac{\delta_f \pi_f^{NE}}{1-\delta_f}$
6	Evaluate ranges of δ_f needed to sustain collusion under 'grim' strategies.

In order to perform a coordinated effects merger simulation, the only 'new' element that must be actively computed is the defection payoff, $\pi_f^{defection}$. Table 4 reports the defection payoffs for our numerical example. They are comparable directly to the numbers reported in Table 2 where static Nash equilibrium profits π_f^{NE} and collusive profits $\pi_f^{collusion}$ were reported. For completeness, note that in the case of a cartel, the total payoff is 270.8 so that each single product firm in this symmetric situation earns the payoff, $\pi_f^{collusion} = \frac{270.8}{6} = 45.1$.

TABLE 4 **This table reports the calculated defection payoffs for each firm under each market structure in our numerical example. Payoffs are shown when firm 1 is the defecting firm.**

	Market structure:					
Firm	*(1,1,1,1,1,1)*	*(2,2,2)*	*(3,3)*	*(4,2)*	*(5,1)*	*6(Cartel)*
1	70.5	128.5	174.5	210.0	238.3	270.8
2	35.0	52.0	57.1	31.2	19.7	
3	35.0	52.0				
4	35.0					
5	35.0					
6	35.0					

Having calculated each of the required objects from the stage game,

[7]Some authors prefer to put payments at the end of periods whereas here they are implicitly placed at the beginning of the period.

$(\pi_f^{collusion}, \pi_f^{NE}, \pi_f^{defection})$ for each firm f we can progress to an evaluation of the incentives for collusion captured in our key formula:

$$V_f^{collusion}(\delta_f) > V_f^{defection}(\delta_f) \iff \frac{\pi_f^{collusion}}{1-\delta_f} > \pi_f^{defection} + \frac{\delta_f \pi_f^{NE}}{1-\delta_f}$$

4.1 A first coordinated effects merger simulation

We begin with the returns to tacit collusion for the single-product firms in our numerical example where market structure is (1,1,1,1,1,1) and the situation following *three* mergers so that the market structure becomes (2,2,2). The advantage of choosing to look at three mergers in this example is that, both before and after the merger, all firms are symmetric. That, in turn, facilitates presentation of the results. Table 5 reports the returns to collusion and also the returns to competition in the numerical example. The shading in the table shows the larger side of the incentive compatibility constraint for each market structure.

First notice that collusion is easier to sustain at higher discount factors for any given market structure; within each market structure the shading is on the collusion side at higher discount rates. Second, notice that in this example, the set of discount factors which can sustain collusion gets broader. If we define a critical discount factor above which collusion is sustainable for any given market structure, in this example we can write $\delta^*_{(1,1,1,1,1,1)} > 0.6 > \delta^*_{(2,2,2)}$.

TABLE 5 **The returns to collusion and competition under the two symmetric market structures**

	(1,1,1,1,1,1)		(2,2,2)	
δ	$V^{Collusion}$	$V^{Competition}$	$V^{Collusion}$	$V^{Competition}$
0	45.1	70.5	90	128
0.1	50.1	73.7	100	136
0.2	56.4	77.7	113	144
0.3	64.4	82.9	129	156
0.4	75.2	89.8	150	171
0.5	90.2	99.4	180	192
0.6	112.8	113.8	226	224
0.7	150.4	137.8	301	276
0.8	225.6	186.0	451	382
0.9	451.2	330.4	902	699
0.99	4512.4	2929.0	9025	6405

4.2 Asymmetry and coordinated effects

In this subsection I present an example of a situation where concentration actually *reduces* the likelihood of collusion. This is consistent with the recent theoretical literature which has suggested that coordination may be harder to sustain under asymmetry; see for example the references and discussion in the very nice survey paper provided by Ivaldi et al (2003).

Table 6 reports the results for two asymmetric mergers. In particular, consider mergers from the baseline market structure (4,1,1) to the alternate market structures (5,1) and (4,2) respectively. This could, for example, be the choice facing an antitrust authority when deciding whether to allow a small firm to merge with its larger or similar sized competitor. In this example, the left hand panel of the table reports that, with a market structure (4,1,1), collusive equilibria can be sustained with discount factors above approximately 0.8; each small firm and the large firm are both willing to support collusion above this point. However, notice that the columns comparing the two small firms' incentives to collude relative to their incentive to cheat, clearly suggest that they are willing to cheat for a far wider range of discount factors than the firm producing four products. Thus, it is the small firms' incentive compatibility constraints which bind in sustaining the collusive equilibria in this case. This observation is central to understanding the incentives to collude following the merger. In particular, when deciding whether to defect, it is the single product firms who have the incentive to free-ride most on multi-product rivals for essentially the same reasons as induces them to cut prices most in the unilateral effects situation.

The first merger examined is the merger of the two small firms, so that the market structure goes from (4,1,1) to (4,2). In that case, the range of discount factors that can sustain collusion changes little. In fact, the only substantive change is in the range of discount factors that will provide the large firm with an incentive to collude which increases from around 0.4 to around 0.5. This however is not the binding incentive compatibility constraint since it is the small firm who has the most to gain by defecting from a collusive arrangement and selling as much as it can.

TABLE 6 **Reports a coordinated effects merger simulation from the initial market structure (4,1,1) to two alternate market structures (4,2) and (5,1). The first merger has little impact on the range of discount factors likely to sustain collusion while the latter makes collusion far *less* likely after the merger because the merger increases market asymmetries.**

	(4,1,1)				*(4,2)*				*(5,1)*			
δ	*Firm with 4 products IC constraint*		*Firms with 1 product*		*Firm with 4 products*		*Firm with 2 products*		*Firm with 5 products*		*Firm with 1 product*	
	Coll.	*Cheat*	*Coll.*	*Cheat*	*Coll.*	*Cheat*	*Coll.*	*Cheat*	*Coll.*	*Cheat*	*Coll.*	*Cheat*
0.0	181	210	45	71	181	210	90	128	226	238	45	71
0.1	201	225	50	75	201	225	100	137	251	259	50	76
0.2	226	243	56	80	226	245	113	148	282	285	56	83
0.3	258	267	64	87	258	270	129	162	322	319	64	92
0.4	301	299	75	96	301	303	150	180	376	364	75	103
0.5	361	343	90	108	361	349	181	206	451	427	90	120
0.6	451	410	113	127	451	419	226	245	564	521	113	144
0.7	602	521	150	159	602	534	301	310	752	678	150	185
0.8	903	743	226	222	902	766	451	439	1,128	992	226	266
0.9	1,805	1,410	451	412	1,805	1,461	902	827	2,256	1,935	451	511

The second merger creates a more concentrated but also more asymmetric market structure since it creates one very large firm owning five out of six of all of the products sold in the market. Following this merger, the ability to sustain collusion actually breaks down for *all* possible discount factors! Simply, the small firm's payoff from participating in the collusive arrangement, given its' very narrow product line, is too small relative to its incentive to deviate. Indeed, its payoff in the Nash equilibrium is actually higher than its payoff under collusion since (i) in the Nash equilibrium the small firm produces far more than under collusion while (ii) the large firm has incentives not to price too low because in doing so it must charge low prices on all its products.[8] For the small firm, in this instance, the incentive compatibility constraint is *never* satisfied since:

$$V^{Collusion}_{SmallFirm} = \frac{45}{(1-\delta)} < 71 + \frac{\delta 49}{1-\delta} = V^{Cheating}_{SmallFirm}, \text{ for all } \delta .$$

These results, for example, suggest that mergers which involve the removal of a small firm, which a merger authority might consider to be a ‘maverick’ firm, one that is an impediment to effective coordination, need not make tacit coordination easier. In fact, by creating further asymmetries in the market, the removal of a small rival may even make it harder to collude.

Note that this result, while similar in flavor to Compte, Rey and Jenny (2002), in the sense that asymmetry makes collusion harder, is rather different in character. In their model, capacity constraints determine both the ability to punish and in particular the temptation to cheat. These two effects mean that the small firm has both less ability to punish the large firm and the large firm has more ability to cheat. Thus, in their model, it is the large firm that will be more tempted to cheat. That is in contrast to the example provided above where it is the small firm who free-rides on the fact that the large firm will set relatively high prices even in the Nash equilibrium while the small firm’s share of collusive profits is relatively small.

One reason this numerical example seems important is that it demonstrates that, in a general asymmetric differentiated product setting, the conditions required for a ‘folk theorem’ result do not hold—at least under grim strategies. As such, the differentiated product collusion game appears substantively different to the homogeneous product game typically studied. In particular, it is important to note that the intuition derived from the homogenous product results provided by Friedman (1971) does not universally extend to the differentiated product context.

To finish let me note that, in our example, the results in Table 2 demonstrate that the post merger payoff to the largest firm in the static (one period) game is 139 under the (4,2) market structure but 188.5 under the (5,1) post-merger market structure. Thus the merger of (4,1,1) to (5,1) would likely generate far greater unilateral effects concerns than the merger to (4,2) and so our particular simulation exercise suggests the following conclusion: This merger investigation should focus on the potential unilateral effects of allowing the merger rather than the potential for coordinated effects.

5. Conclusions

This paper has shown how to perform merger simulations to evaluate the presence of ‘coordinated’ effects of mergers. In doing so we extend the methods developed for the evaluation of mergers beyond the unilateral effects case. The method proposed here is entirely consistent with collusion theory and only requires a small additional amount of effort on the part of the agency or researcher relative to performing a unilateral effects

[8]Recall from TABLE 2 that in a collusive arrangement the payoff to each single product firm is 270.8/6 = 45 (column headed cartel) while the Nash equilibrium payoff to the single product firm in the (5,1) market structure is 49. (See payoff to firm 2 in the (5,1) column.) The reason is that there’s a large ‘free-riding’ effect available to the small firm in the (5,1) static Nash equilibrium.

merger simulation. We believe the tools and techniques developed here will be useful for policy makers, providing a set of practical tools for evaluating the coordinated effects of mergers.

We provide analytic results that are useful for a large class of demand structures and arbitrary ownership structures. However, by considering a numerical example, we have avoided a number of practical issues that would need to be addressed before applying these tools into the field.

First, in practical settings, we must determine the length of a ‘period.’ Clearly, a period is appropriately defined as the amount of time that it would take for rivals to detect a pricing deviation and change their prices appropriately, which under grim strategies would mean reverting to the Nash equilibrium. We believe it will usually be possible to infer an appropriate period length on the basis of the observed historical frequency of price changes and timeliness of rival responses to news in a specific application, though deciding on an appropriate period length may require some judgment.

Second, we must determine the sequence of future payoffs under both collusion and competition. Here, we choose to follow the theoretical literature which treats the subsequent games as identical and this is likely a sensible approach in many instances. In others it may be appropriate to assume that markets will grow at some rate over time. Either method makes strong assumptions, including the fact that we assume an absence of entry, product sets do not evolve and so on. Such assumptions, though strong, are of course identical to those made both by the unilateral effects merger evaluation literature and by investigators progressing without describing their understanding of the applicable model of the industry. Provided the results are interpreted with appropriate caution, and some sophistication, merger simulation provides a useful set of techniques to help in the evaluation of both the unilateral- and now coordinated-effects of mergers.

This appendix develops the matrix expressions useful when computing Nash equilibrium prices. The results are provided under the behavioural assumption that each firm prices its products to maximize its profits from the stage game but are sufficiently general to cope with an arbitrary ownership structure.

We saw in the text that any firm *f*'s first order condition may be written as:

$$D_k(p)+\sum_{j=1}^{J}\Delta_{jk}\frac{\partial D_j(p)}{\partial p_k}(p_j-c_j)=0 \text{ ie, } a_k+\sum_{j=1}^{J}b_{kj}p_j+\sum_{j=1}^{J}\Delta_{jk}b_{jk}(p_j-c_j)=0 \quad (1)$$

Notice that there are one of these first order conditions from firm *f*'s objective function for every $k \in \mathfrak{I}_f$. Since every product $k \in \mathfrak{I}$ is owned by some firm, we obtain a total of J first order conditions—one for every product being sold. We may then 'stack up' the first order conditions. To do so it's useful to introduce some matrix notation. Define the (Jx1) vectors $a=\begin{bmatrix} a_1 \\ \vdots \\ a_J \end{bmatrix}$ and $c=\begin{bmatrix} c_1 \\ \vdots \\ c_J \end{bmatrix}$ and also the (JxJ) matrices:

$$B'=\begin{bmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1J} \\ \vdots & & \vdots & & \vdots \\ b_{k1} & \cdots & b_{kj} & \cdots & b_{kJ} \\ \vdots & & \vdots & & \vdots \\ b_{J1} & \cdots & b_{Jj} & \cdots & b_{JJ} \end{bmatrix} \text{ and } \Delta \bullet B=\begin{bmatrix} \Delta_{11}b_{11} & \cdots & \Delta_{j1}b_{j1} & \cdots & \Delta_{J1}b_{J1} \\ \vdots & & \vdots & & \vdots \\ \Delta_{1k}b_{1k} & & \Delta_{jk}b_{jk} & & \Delta_{Jk}b_{Jk} \\ \vdots & & \vdots & & \vdots \\ \Delta_{1J}b_{1J} & \cdots & \Delta_{jJ}b_{jJ} & \cdots & \Delta_{JJ}b_{JJ} \end{bmatrix}.$$

where $\Delta \bullet B$ is known as the 'Hadamard' or element by element matrix product'.

We may then 'stack up' the first order conditions to obtain:

$$\begin{bmatrix} a_1 \\ \vdots \\ a_k \\ \vdots \\ a_J \end{bmatrix}+\begin{bmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1J} \\ \vdots & & \vdots & & \vdots \\ b_{k1} & \cdots & b_{kj} & \cdots & b_{kJ} \\ \vdots & & \vdots & & \vdots \\ b_{J1} & \cdots & b_{Jj} & \cdots & b_{JJ} \end{bmatrix}\begin{bmatrix} p_1 \\ \vdots \\ p_j \\ \vdots \\ p_J \end{bmatrix}+\begin{bmatrix} \Delta_{11}b_{11} & \cdots & \Delta_{j1}b_{j1} & \cdots & \Delta_{J1}b_{J1} \\ \vdots & & \vdots & & \vdots \\ \Delta_{1k}b_{1k} & & \Delta_{jk}b_{jk} & & \Delta_{Jk}b_{Jk} \\ \vdots & & \vdots & & \vdots \\ \Delta_{1J}b_{1J} & \cdots & \Delta_{jJ}b_{jJ} & \cdots & \Delta_{JJ}b_{JJ} \end{bmatrix}\begin{bmatrix} p_1-c_1 \\ \vdots \\ p_j-c_j \\ \vdots \\ p_J-c_J \end{bmatrix}=0$$

The solution to this set of equations, $p^{NE}=(p_1^{NE},...,p_J^{NE})$, provides the prices at which each firm is maximizing its profits given the prices of others, and hence is the Nash equilibrium price vector to the stage game. The vector of first order conditions may be written more compactly in matrix terms as $a+B'p+(\Delta \bullet B)(p-c)=0$. Rearranging, we get $(B'+(\Delta \bullet B))p=-a+(\Delta \bullet B)c$ and hence Nash equilibrium prices can be computed using the matrix formula: $p^{NE}=(B'+(\Delta \bullet B))^{-1}\left(-a+(\Delta \bullet B)c\right)$.

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